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The Encoding-Error Model of Pathway Completion without Vision

We present a model that accounts for errors in short-cutting to complete a triangular pathway by individuals deprived of visual input. The model assumes that systematic error arises from components of navigation concerned with encoding an internal representation of the pathway, rather than the computation of a homeward trajectory or motor output per se. Subjects' tendency to compress the range of actually produced turns and distances, in comparison to the range of correct values, is attributed to regression toward the mean of encoded values during encoding of segments and turns, in the face of uncertainty about the actual values. Individual-subject variations are attributed to differences in the encoding-function parameters, not to variations in the processes themselves. The model provides excellent accounts of data obtained with triangular pathways but fares less well when pathway complexity increases, at which point errors do not appear to be solely attributable to encoding processes. The sources of error identified by the model are likely to play a role in navigation more generally.

Processes involved in sensing, representing, and recognizing environmental information and using that information to navigate have been the long-standing interest in cognitive psychology and geography (for example, Golledge 1992). An understanding of these abilities is critical to the effective design of systems intended to aid navigation, such as Geographic Information Systems, electronic maps, or hand-held GPS systems (in the nascent stage for personal navigation). The same understanding is sought by today's architects and urban planners, who are sensitive to the fact that people's navigation behavior is mediated by their mental models of physical environments (Passini 1984).

Two general concerns in the navigation literature are how features of the environment are used to plan routes and guide ongoing navigation and how the act

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of navigating through an environment enables one to learn about it. The answers to these questions are considerably different for individuals who are navigating with and without sight. The sighted traveler can take advantage of a variety of external cues to spatial layout, such as visible texture gradients, remote landmarks, and optical flow patterns during movement. These cues can directly guide way-finding, for example, by allowing people to home in on a destination, and they provide a rich information source for the encoding of a survey representation of the environment into memory. In contrast, an individual traveling without sight has a very different set of primitive cues, primarily acceleration information provided by the vestibular system and kinesthesis from moving limbs (although tactual information about the terrain underfoot or estimates of exertion may prove useful adjuncts). By examining human navigation under these more limited sensory circumstances, we hope to understand basic components of spatial information processing that also play a role in sighted travel.

In the absence of positional information provided by sight (or occasionally, by other modalites such as audition), an organism must navigate using either inertial navigation or dead reckoning (Schiesser 1986). These are inclusively referred to as path integration. Inertial navigation refers to sensing self-acceleration (by accelerometers or inner-ear mechanisms) and performing double integration to obtain change in position (for example, Potegal 1982). Dead reckoning is a form of navigation in which the traveling organism senses self-velocity (heading and speed) and integrates to obtain position within some coordinate system (Mittelstaedt and Mittelstaedt 1982). Although dead reckoning does not directly rely upon external position information, such as optical, acoustic, chemical, or radio signals, such signals may be involved in the sensing of self-velocity.

Judging from published reports, a number of nonhuman species have uncanny path integration ability (Mittelstaedt and Mittelstaedt 1982; von Saint Paul 1982; Muller and Wehner 1988). For example, von Saint Paul (1982) provided compelling evidence that goslings were able to integrate visual information about passive self-motion while in a cart to determine displacements from home, as shown by their walking the appropriate distance and direction on the homeward leg upon their release. The possibility that the goslings were using position information (that is, landmarks) specifying the location of home was ruled out by a condition in which the sides of the cart were covered during parts of the traverse. The goslings completely ignored these portions of the traverse, setting out in the direction determined by the integrated uncovered path, even if home was in the opposite direction. The same finding suggests that the goslings used some form of dead reckoning rather than making use of acceleration information, which was available in both the covered and uncovered portions of the traverse. For an excellent review of path integration and cognitive mapping in animals, see Gallistel (1990).

As yet, we know little about path integration in humans under conditions where the observer has access to visual information about self-velocity but not about position. However, considerable work has been done on navigation by human adults and children where visual information is completely lacking (for example, Cratty 1965; Dodds, Howarth, and Carter 1982; Klatzky et al. 1990; Landau, Gleitman, and Spelke 1981; Loomis et al. 1993; Rieser, Guth, and Hill 1986; Sadalla and Montello 1989; Worchel 1951; Yamamoto 1991). Much of this work investigates whether navigation ability depends upon the extent of prior visual experience, by comparing sighted, blindfolded subjects to congenitally and adventitiously blind groups.

In this paper we develop a model that accounts for systematic errors in path integration carried out without visual input. By systematic errors, we mean behavioral errors that stem from a systematic distortion in processing. They are assumed to derive from internal processes common to a substantial portion (if not all) of a tested population. We distinguish systematic errors from random behavioral errors resulting from stochastic (noise) processes that vary both within and between individuals. We are concerned with systematic error at the group level and not stable but idiosyncratic response tendencies of an individual (for example, degree and direction of veer when attempting to maintain a heading).

A substantial number of systematic error tendencies have been described when information is encoded from maps or derived from sighted locomotion through a space (see MacEachren 1992 and Tversky 1992 for review). For example, spatial regions tend to be remembered as aligned with major reference axes (Lloyd 1989a; Tversky 1981); ratings of differences in distances from a reference point are greater for locations closer to the referent than those far from it (Holyoak and Mah 1982); and filled spaces are perceived to be larger than empty ones (Thorndyke 1981). In distance estimation tasks, a systematic pattern of overestimating short distances and underestimating long ones has been found for stimuli as diverse as maps (Lloyd 1989b; MacEachren 1992), patterns traced with the finger (Lederman, Klatzky, and Barber 1985), and, as will be discussed further below, locomoted routes (Loomis et al. 1993).

Our specific interest is in errors in a pathway-completion task, in which individuals followed a pathway of linear segments and turns and then attempted to return directly to the origin; both initial exposure to the pathway and the completion occurred without vision. The data derived from experiments of Loomis et al. (1993) and Klatzky et al. (1990). Loomis et al. found that blind and sighted, blindfolded subjects compressed the range of responses relative to the correct values. That is, when subjects were supposed to produce large values of turn or distance, they produced lesser values by terminating the response prematurely, and conversely, when they were to produce small values, they tended to overshoot. On the whole, the responses showed substantial regression toward the mean of actual values. Klatzky et al. found that completion errors increased with the complexity of the path, as defined by the number of segments and the geometric pattern.

We adopt a five-component framework (from Loomis et al. 1993) for the processes underlying performance in tasks of this sort. The components are sensing, creating a trace of the route, forming a survey representation, computing desired trajectories, and executing those trajectories. The present model attributes the systematic errors observed in pathway completion to the initial components, those that culminate in forming a survey representation.

That subjects approximately converge on the origin of locomotion when trying to complete a pathway indicates that they are using a survey representation for the task. Indeed, the task demands are such that a route representation alone would not be sufficient. However, a survey representation can contain different levels of detail. At one extreme is a minimal metric representation that indicates only observer-relative location of the origin, in terms of the turn and distance that would be needed for the observer to return home (Fujita et al. 1990; Muller and Wehner 1988). These parameters constitute what has been called the "homing vector" (Fujita et al. 1990), which is updated with each step of the observer. Such a representation is history free, for it maintains no record of the path that the observer took to arrive at the current location. Evidence against the history-free hypothesis comes from data reported by Loomis et al. (1993): The time to initiate pathway completion increased with the complexity of the previously traversed route. If subjects had been continuously computing the homing vector and discarding information about the path, the latency to start homeward should have been independent of pathway complexity. In light of this result, we assume that subjects employ a representation that provides information about their outbound paths, not just a homing vector.

We use the term "encoding" to refer to the set of processes leading to the internal representation of the navigated area that mediates the response; this term subsumes the first three components of our general framework described above. Description of the encoded representation is sufficient, in the present model's view, to characterize systematic trends in subjects' performance. That is, it assumes that encoded values are fed into a computational process that introduces no further systematic error, and that the computed trajectory is also executed without systematic bias (at least on average). For this reason it is called the *encoding-error* model.

The assumption of no systematic error in computing the homeward trajectory is consistent with an internal symbolic process akin to cognitive trigonometry. Another possibility is measurement of a mental image, analogous to perceptually assessing angles and segment length viewed in a map. In contrast, systematic error in computing the trajectory could result if subjects used weak heuristics with biases. For example, in the task of completing a triangle after traversing two segments and a turn, they might use the rule, "If the initial turn was small, simply turn 180 degrees and go a distance equal to the sum of the two segment lengths; if it was very large, don't turn and go a distance equal to the first segment minus the second." This would result in overestimates of the return distance for triangles with small initial turns and underestimates for those with large ones.

Holyoak and Mah (1982) suggested that even in the presence of accurate knowledge about distances, systematic error can arise from transformations involved in the process of judging them. In particular, compression of distance can result from converting known distances into values on a categorical scale with increasing category widths. Holyoak and Mah attributed this error to a stage in which the distance is retrieved from long-term memory and converted to a new, categorical representation in working memory. This retrieval process has been called "decoding" to distinguish it from the original encoding into long-term storage (Lloyd 1989a, 1989b). However, the term "encoding" is often used for storing information at any level (Klatzky 1980). In this sense, such decoding is a form of encoding (that is, encoding into the working memory) and would be considered part of the present encoding processes because it precedes computation of the response trajectory. (It should also be noted that the distinction between encoding into working memory and encoding into more enduring memory is of limited application to the present task, in which a response is made immediately after exposure to the pathway.)

The encoding-error model also assumes that systematic errors in pathway completion do not result from the last component of our conceptual framework, execution. There is evidence for this assumption from a number of studies in which subjects first viewed a target and then walked to it without vision. These studies showed virtually no systematic error (Elliott 1987; Loomis et al. 1992; Rieser et al. 1990; Thomson 1983). Because vision was available in these studies for encoding the pathway and for the minimal computation required to plan the trajectory, errors at these stages should have been minimal, and those observed could be substantially attributed to the output stage. Hence these cases provide some assessment of "pure" output error in walked distance, and they are inconsistent with the systematic sharp compression of distance found in the Loomis et al. pathway completion task.

As was noted above, the present paper describes the application of the encoding-error model to the data from Loomis et al. (1993) and from Klatzky et al.

(1990). In both studies, subjects were led along pathways of linear segments with turns between each segment, after which they attempted to return directly to the origin of locomotion. By means of video cameras mounted along two sides of a work space about fifteen meters square, a subject's position was recorded at a rate of approximately six hertz, allowing the return trajectory to be determined. The data comprise the subject's turn from the terminus of the pathway back toward the origin and the return distance walked. The model is fit both to group means and to data from individual subjects. Although we model data obtained from subjects navigating without vision, the model makes no assumption about the nature of the original navigation, other than that it is subject to distortion and noise.

THE ENCODING-ERROR MODEL

The model has four underlying assumptions: (1) The internal representation satisfies Euclidean axioms. (2) The length of a straight-line segment is internally encoded by a function that provides a single encoded value of segment length for each stimulus value. Thus two straight paths of equal length will have equal encoded length values. (3) The value of a turn angle is internally encoded by a function providing a single encoded value for each stimulus value (thus, turns of equal magnitude are encoded as equal). (4) There is no systematic error in either the computation of the homeward trajectory or its execution.

Given these assumptions, our approach can be illustrated simply by the case where the subject walks two segments of equal length with a ninety-degree turn between them, forming (with a completing segment) a right isosceles triangle. The subject then tries to walk homeward, and his or her turn and distance are measured. If the subject's internal representation is of an isosceles triangle (as dictated by assumption 2 above), we have the geometric constraints that (a) the two remaining angles must be equal, and (b) all three angles must sum to 180 deg. Further, we have an estimate of the value of the two equal angles; it is determined by the turn that the subject actually made in returning to the origin. The two constraints, together with the turn response, are sufficient to derive the remaining angle. For example, if the subject turns forty-six degrees, the right angle must be internally represented as eighty-eight degrees (that is, $180 - 2 \times 46$). The sides of the represented triangle, and hence its scale, can be determined by the subject's walked distance homeward, which is an estimate of the third (base) leg.

Using this approach to analyze the results of three right-isosceles cases from Loomis et al. (1993), it appears that the subject's representation of the shape of the triangle is in fact quite accurate, as shown in Table 1. The derived values of scale also suggest accurate representations of the shortest values of segment length, with underestimation increasing as the size of the triangle increases.

Alternative conceptions of the internalized triangle are possible because the system is underconstrained by the data. For example, the subject might encode the initial turn correctly, but not perceive the shape to be isosceles. In this case, the constraints of the data and the assumption of a correct value of turn would be

TABLE 1
Average Observed Responses and Derived Values for the Equal Sides and the Right Angle of the Three right Isosceles Triangles Used in the Pathway Completion Task. Scale = ratio of derived side length to actual side length.

Average turn response	Average distance response	Derived right angle	Derived side length	Actual side length	Scale
44	272	92	189	200	.94
46	476	88	342	400	.86
47	603	86	442	600	.69

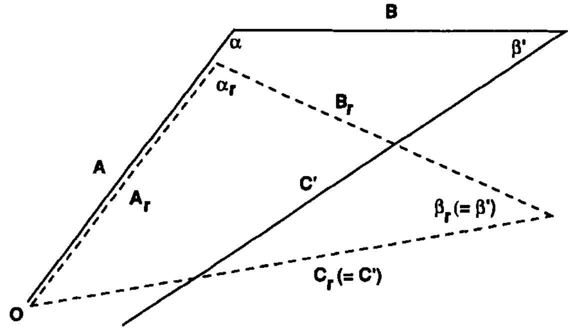

FIG. 1. Geometry of the Triangle-Completion Task. See text for explanation.

sufficient to derive the shape of that triangle. However, our assumption (see above) is that the subject encodes the two equal legs as being equal, rather than that he or she perceives the angle between them without error.

We presented the example involving isosceles triangles to give the flavor of the model. At this point, we describe its assumptions in detail. Initially, we restrict ourselves to the simple case where the pathway has two segments with a turn between them, as illustrated in Figure 1. At the end of the pathway, the subject should make a turn toward the origin and return along a third segment, thus completing a triangle. We will call the segments A, B, and C, respectively. The turns will be referred to by the inner angles of the triangle α (180 degrees minus the physical turn between segments A and B) and β (180 degrees minus the physical turn needed to head toward the origin). The data available from the subject's performance are the inner angle formed by the subject's actual turn toward the origin and the actual return distance walked, which we will term β' and C'.[1] Given A, B, and α, which are controlled by the experimenters, and β' and C', which are produced by the subject, we wish to derive the subject's encoded representation of the walked pathway.

Shown in Figure 1 are the actual stimulus values of A, B, and α. (Although C is not shown in the figure, it is the implicit line directly connecting the terminal points of A and B.) Note that the origin, O, is the point where segment A initiates and segment C would terminate. Also shown is a hypothetical subject's response: a turn and traversed distance (β' and C'). Superimposed on the stimulus is the hypothesized internal representation of the subject. Initially, we know only two values of this configuration: the inner angle at the point of turn toward the origin (β') and the third segment back to the origin (C'). We wish to know the encoded values of A, B, and α, namely, A_r, B_r, and α_r, (where the subscript r refers to the subject's representation and, where appropriate, the observed response).

The subject intends the response to terminate at the true origin, although it generally terminates at a different endpoint because of errors in encoding the stimulus. Without loss of generality, we assume, as shown by the dashed line in

[1]We present the data and model fits in terms of the inner angles for simplicity of graphical presentation and trigonometric formulas. This convention does not affect our reasoning and conclusions. In particular, the amount of regression toward the mean in the subject's turn (perceived, 180 $-\alpha$, or response, 180 $-\beta$), as measured by the slope of a function relating that turn to the actual value, is precisely the same when the inner angle (α or β) is substituted. (To see this, consider that with regression to the mean, the encoded value of a turn that is less than the mean will increase. The corresponding encoding of the inner angle must then decrease because the two angles sum to 180, implying that the inner angle must move toward its mean as well.) In contrast to the constancy of slope, the intercept of the function relating the subject's turn to the actual value does change, by the following formula: Intercept for inner-angle function = 180(1 − slope) minus intercept for outer-angle function.

Figure 1, that the subject's encoded triangle has its first segment aligned with stimulus segment A and that the return endpoint coincides with the origin, as the subject intended. Then the observed C′ and β', by the assumption that there are no errors in execution, are equal to their internally represented values (C_r and β_r). These in turn constrain the values of the unknown parameters (A_r, B_r, and α_r), but not uniquely. When specific values are adopted for any set of such parameters, the subject's endpoint can be predicted. The distance between this predicted endpoint and the subject's actual endpoint provides a goodness-of-fit measure for the given parameter set.

For only one triangle, of course, parameter values can be chosen that provide a perfect fit to the subject's data, but many such parameter sets are possible. However, the parameter sets are constrained when we consider that the subject is assumed to encode pathway lengths and angles in the same way for all stimulus configurations. Thus, our goal is to fit a set of configurations with a common set of parameters, representing the encoded values for the segment lengths and angles used to construct the physical stimuli. When the correspondence between each actual stimulus value and the estimated encoded value is established for some set of segment lengths and turns, the underlying "encoding functions" can be derived. The encoding function is an equation with as few parameters as possible that adequately captures the established correspondence between a set of stimulus values and their encoded values.

Parameter estimation is accomplished by a least-squares technique. At each step, potential encoded values for the segment lengths and turns used in constructing the physical stimuli are assigned. For a given stimulus configuration, representing some combination of A, B, and α, the corresponding assigned encoded values A_r, B_r, and α_r are used to derive a C_r and β_r, using trigonometric equations. That is, it is assumed that the subject's internal representation of the triangle satisfies theorems of trigonometry, for example, the law of sines. The predicted values then determine the x, y coordinates of the predicted endpoint. Its distance from the subject's actual endpoint (that is, the model error) for that configuration is then computed. This process is performed for all physical configurations using the same parameter set, and squared errors are summed. Parameters that minimize total squared error are sought in an exhaustive search over a plausible range. Derivation of the equations is shown in Appendix 1.

FITTING AVERAGE RESULTS

Triangle Completion

Our initial application of the model is to data from Loomis et al. (1993). They tested three groups of adult subjects—sighted, congenitally blind, and adventitiously blind—who were approximately matched in terms of age and educational level. The blind subjects had sufficient navigation skills to allow them to find their way in their communities. The subjects took part in a series of small-scale spatial tasks (for example, mental rotation of hand-held figures), simple locomotion tasks (for example, maintaining a heading or repeating a walked distance), and complex locomotion tasks. Of the latter, we focus on the task of completing a triangle—returning directly to the origin after walking two legs with a turn between them. Twenty-seven triangles, representing a factorial combination of three initial leg lengths (A—2, 4, or 6 meters), three second-leg lengths (B—2, 4, or 6 meters), and three angles between the legs (α—60, 90, or 120 degrees), were used. The subject performed on all twenty-seven pathways in succession. The resulting data are the subject's distance and turn values when attempting to return to the origin

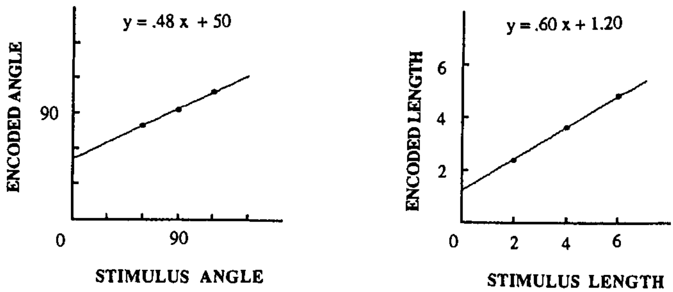

FIG. 2. Functions Relating the Hypothetical Encoded Values for Each Value of Stimulus Angle (α) and Distance to the Actual Values. The best-fitting linear equations are also shown.

of a walked configuration. Our interest at this point is in fitting the mean values of these responses, which we treat as stable estimates of a typical subject's internal representation of the pathways, relatively free from stochastic elements or individual idiosyncracies. (In support of this assumption, we found the distributions of both distance and turn errors over subjects to be essentially normal, as would be expected if they were the result of random variations around a stable mean value.)

The parameter values derived for the average data are shown in Figure 2 (left panel: segment lengths A or B; right panel: α). As can be seen in Figure 2, a linear function describes well the relationship between the parameter estimates and the actual values for both length and angle.[2] That is, there is a linear function relating the three estimated values of encoded segment length to the actual values, which we will refer to as the length-encoding function. The angle-encoding function, which relates the estimates of the encoded values of α to the actual values, is also linear. Because the values encoded for the stimuli are linearly related to the actual values, the number of parameters needed to specify encoding drops from six, the number of turns and angles represented over the set of physical stimuli, to four, namely, the slopes and intercepts of the two encoding functions. (Henceforth, when we refer to encoded values, we mean the values predicted by the encoding functions, rather than the parameter estimates used in the fitting of those functions.)

The slopes of the encoding functions are substantially less than 1, with the result that the range of encoded values is compressed relative to the range of actual values. Moreover, the mean of the encoded values is approximately the mean of the actual values, so that the encoding process as a whole demonstrates regression to the mean. It is this regression that produces the compression of range in the predicted (and observed) responses. Later on we will consider why this regression to the mean might occur.

Figure 3 shows the degree of fit of the model by depicting the predicted errors in turn and distance as a function of the empirically obtained errors, for each of the twenty-seven configurations. We plot errors rather than response values, because even a noisy prediction of the responses would give a high correlation between predicted and observed values. The correlations in Figure 3 are very high (r^2 = .93 and .92, for distance and turn, respectively), and the slopes are close to 1.0 (1.17 for distance and .98 for turn).

[2]The value of this function at α = 180 (that is, turn = 0) is substantially less than 180, which means that subjects encode small turns as having substantial positive values. Because the smallest turn studied here was 60 degrees, the present function is a reasonable one for the range of stimulus values. However, had subjects experienced a turn of zero, it is doubtless that the encoded value would also be zero, implying that the encoding function was nonlinear.

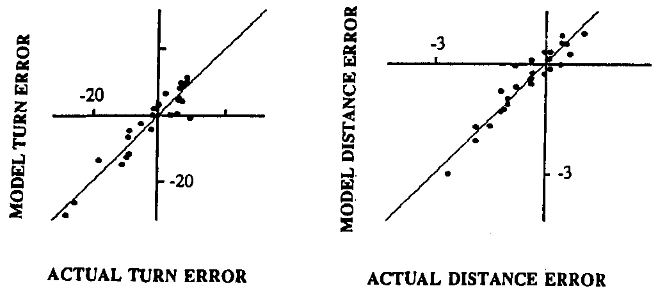

FIG. 3. Errors Predicted by the Model for Turn and Distance as a Function of the Actual Values.

Additional evidence of the success of the model for each configuration can be seen in Figure 4, which shows the location of the endpoint predicted by the model versus that observed for the group of subjects. The close correspondence between data and model is obvious. For each row, the length of segment A is constant; for each column, the value of α is constant; the length of segment B is shown as a varying parameter in each component of the figure. Within each row component,

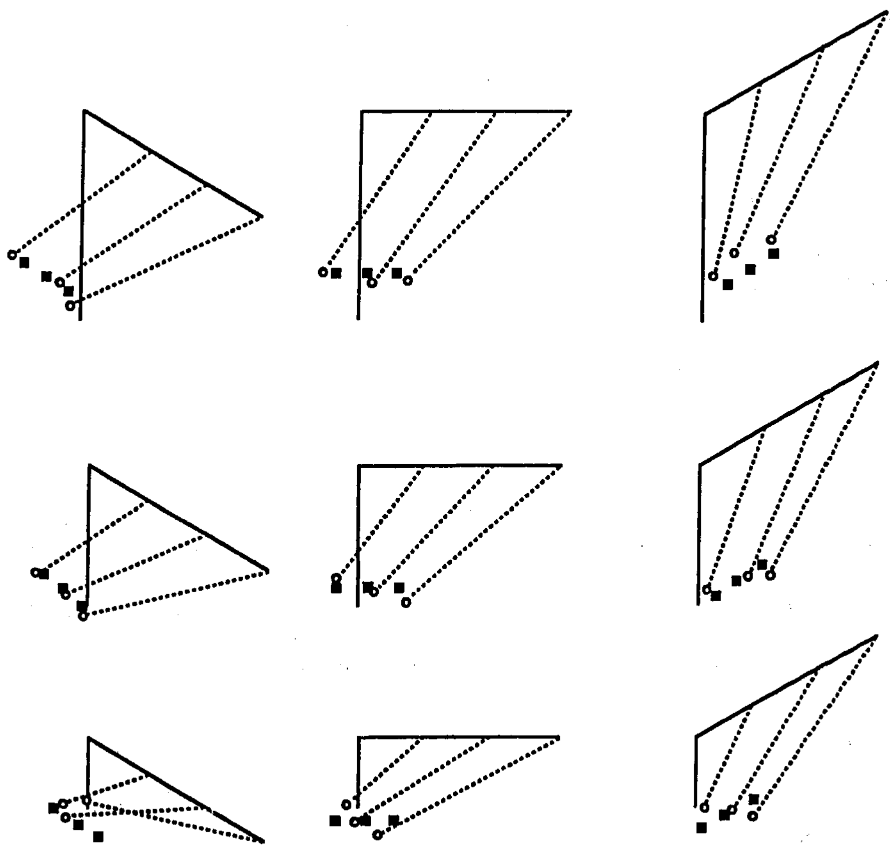

FIG. 4. Endpoints Predicted by the Model (Squares) and the Actual Average Endpoint (Circle), for Each Configuration. In each panel, the length of the first segment and turn angle are held constant, and three configurations, varying in length of the second segment, are depicted.

the predicted values form a straight line, which has the same slope for all components in a column, because the same parameter α applies. The empirical data also show this tendency. (The one exception, row 3, column 1, is accounted for by the tendency of subjects who walked outward from the triangle to leave the workspace surveyed by the video measurement system, thus invalidating their response. As a result, the data are biased toward inward responses.)

Extension to Complex Pathways

The encoding-error model performs impressively in accounting for data from the triangle-completion task. Can it be extended to more complex pathways of three segments? Such pathways were included in the study of Klatzky et al. (1990), where subjects completed paths of one, two, or three segments at two different scales (paths of one segment were simply retraced in the opposite direction). Performance was found to worsen as the number of segments increased; this general effect is predicted by the model, due to increases in the number of inputs that must be encoded. However, we did not expect the model to fare well in predicting the most complex (three-segment) pathways because it is likely that errors in this case reflected sources other than just regression to the mean during input encoding. In one of the pathways, the third segment crossed over the first; this led to extremely variable—seemingly, quite random—responses over the group of subjects, suggesting corresponding variability in processing. Some subjects clearly "got lost," in that they did not realize that the segment had crossed over; as a result, they did not turn back toward the origin but continued onward away from it.

The encoding-error model was applied to these data, and as expected, it did not fare well. For example, even with a fifth parameter representing a constant overturn, the function relating predicted to actual error on the twelve large-scale pathways had an r^2 of .71 for distance and .64 for turn. Accounting for the potential range of factors affecting encoding and computation of the homeward trajectory appears to be beyond the model's scope.

AN OUTPUT-ERROR MODEL

A potential alternative to the encoding-error model is one that attributes the observed systematic error not to encoding the input segments and turns, but to response execution. This model derives from the fact that subjects' mean responses for both distance and turn are a nearly linear function of the correct response (see distance function in Figure 5, bottom). These response functions can be described by four parameters, namely, their slopes and intercepts. The output-error model simply assumes that the functions are accurate models of output error—a systematic tendency to underproduce the desired values of long responses and overproduce short ones during execution. That is, subjects correctly encode segment A, segment B, and α, and compute accurately the requisite segment C and β; all the error in the output is due to compression of the range of response at output.

However, empirical findings argue against this model. First, as noted in the introduction, assessment of errors attributable to the output stage (that is, when circumstances promote accurate encoding) does not show the sharply compressive pattern of the presently considered data.

The output model also fails because of results shown in Figure 5. If one had accurate knowledge of the required homeward trajectory at the start of the response and all systematic error were in output, then pathways that required the same return distance (or turn) should produce the same error. That is, errors should depend only on the navigator's distance (or angle) with respect to the origin

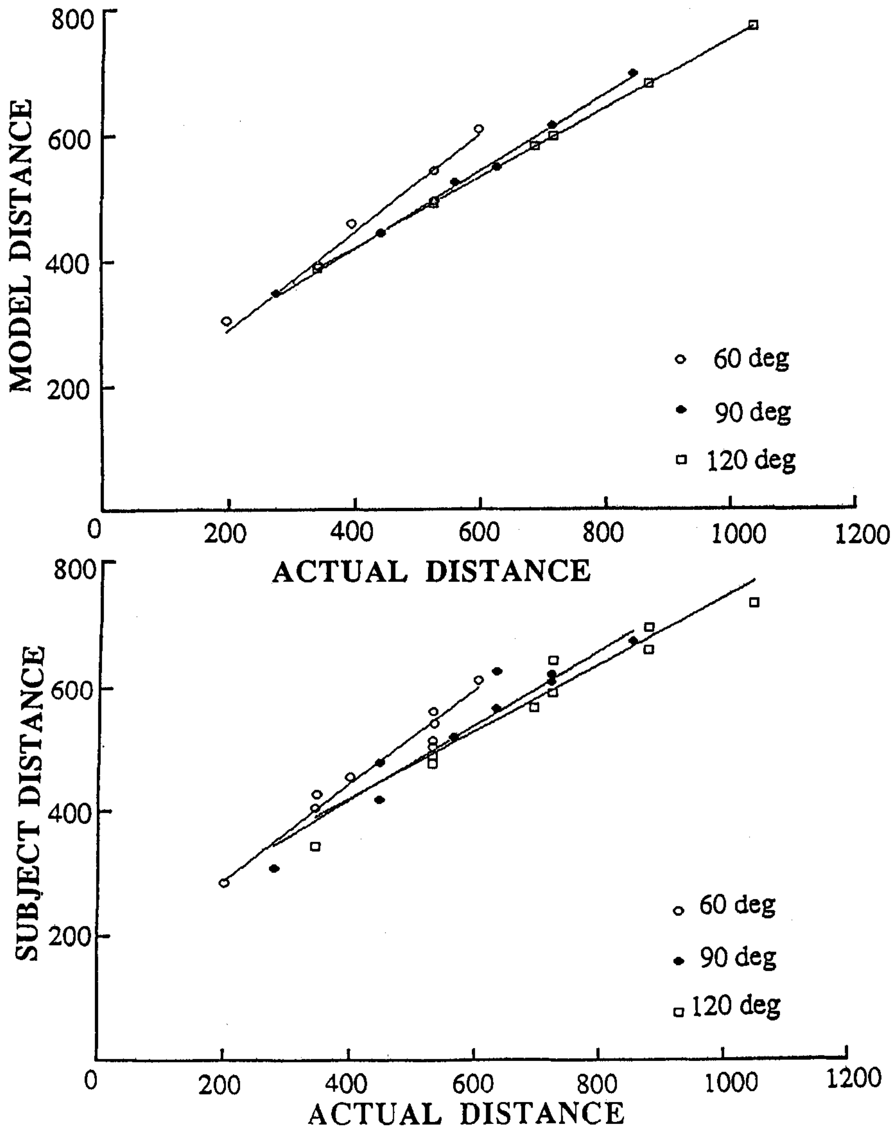

FIG. 5. Top: Function Relating Distance Response Predicted by the Model to the Correct Value, for Each of the Twenty-seven Pathways. Points are marked to indicate the value of the angle α. Bottom: Same as top, but with observed mean distance responses replacing predicted values.

at the time of response, regardless of the pathway that led to that point. Evidence against this prediction can be seen in Figure 5, which shows sensitivity to the stimulus angle α in the Loomis et al. data. The lower panel plots the average response against the correct response for distance, where points are marked with respect to the value of α for the corresponding pathway. As we noted above, the function is nearly linear overall. However, there is a small but clear tendency for lower values of α to lead to increasingly steep functions relating the output response to the correct value. For the same x-axis value (the correct return distance), different responses are observed, depending on the turn within the stimulus. The same phenomenon is predicted by the encoding-error model, as shown in the upper panel (where predictions are based on the same parameters as derived above). The tendency is also observed, but in a less pronounced way, for turn responses (not shown).

FITTING INDIVIDUAL SUBJECT'S DATA

We next consider performance of individual subjects in the Loomis et al. (1993) study. In doing so, we will refer to three sets of functions. One comprises the *encoding* functions for segment length and angle, that is, the model-derived functions relating the encoded values of A (or B) and α to those in the physical stimulus. Each encoding function has three points (that is, there are three segment lengths or three angles) which are fit linearly to obtain a description of encoding in terms of slope and intercept. The second type of function will be referred to as the *empirical response* function; it plots the subject's response as a function of the correct response over the twenty-seven stimulus configurations. There is an empirical response function for each response measure: turn and distance. Functions for a sample of subjects are shown in Figure 6. Finally, the model predicts a response for each configuration; hence there is a *predicted response* function for turn and for distance.

As in the average data, the empirical response functions for individual subjects are generally increasing, indicating some sensitivity to the nature of the pathway. Perfect performance would yield a function with slope of 1. However, our subjects generally show less than 1. That is, their response tendencies are "compressive," in that there is relative overresponding to low values and underresponding to high ones. (However, for slopes less than 1, the intercept determines the value towards which there is regression.) When applied to the average data, the encoding-error model predicted compressive response functions because there was similar compression in the encoding functions for length and angle (that is, there tends to be underestimation of large values and some overestimation of small ones).

To examine individual differences, we fit the model to twelve of the thirty-seven subjects studied by Loomis and associates (1993), four from each of three groups that were classified by visual status (congenitally and adventitiously blind and sighted). Our focus here was on the overall pattern of the response functions and not on responses to individual pathways (which, as will be described below, are subject to random error tendencies that preclude point-by-point modeling of the function). Subjects were selected to represent a range of compression in their empirical response functions. Table 2 presents the slopes, intercepts, and r^2 values for each of the six functions described above (model-derived encoding functions for segment length and angle, empirical response functions for turn and distance, and predicted response functions for turn and distance), by subject.

Consider first the encoding functions (top of Table 2), which tend to be fit well by linear trends. There is considerable variability in the slope, which indicates the degree to which variations in the value of turn angle or segment length are compressed in encoding. Subject Con-5 (congenitally blind) shows a zero slope for angle; this subject essentially interpreted every turn as if it were 120 degrees. On the other hand, Subject Con-8 (also congenital) has an encoding function for length with a slope of 1; this subject encodes distances with virtually no compression of range.

The derived encoding functions were used to predict an individual subject's response on each pathway. We do not test the model's fit on a pathway-by-pathway basis; rather, we consider the extent to which the model predicts the general form of the response function. Table 3, left column, shows the correlations over the twelve subjects between parameters of the predicted response function and the

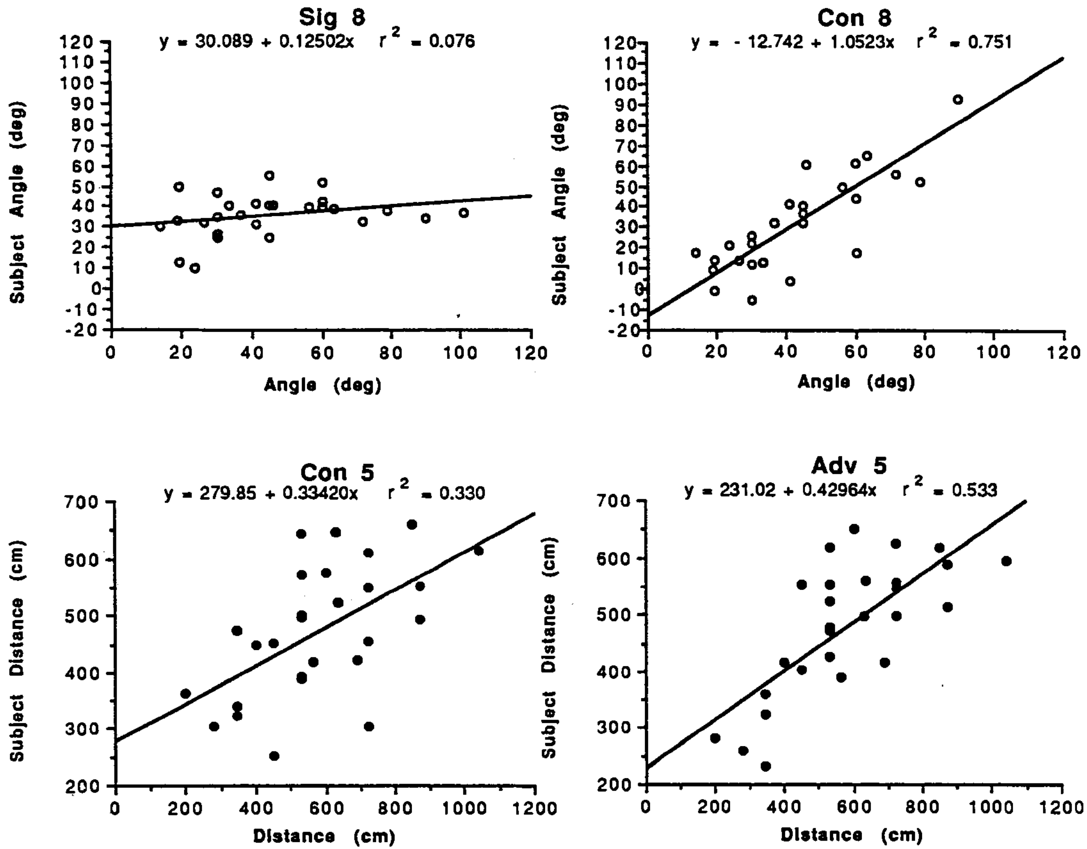

FIG. 6. Function Relating Observed Distance or Turn-angle (β) Response to the Correct Value, over the Twenty-seven Pathways, for four cases.

TABLE 2

Slope (S), Intercept (I), and r^2 for the Encoding Functions for Length and Angle, Empirical Response Functions for Turn and Distance, and Predicted Response Functions for Turn and Distance (without Gaussian Noise), by Subject. The subjects are labeled with the same numbers as in Loomis et al. (Con = congenital, Adv = Adventitious; Sig = Sighted).

	Encoding, Angle			Encoding, Length		
Subject	S	I	r^2	S	I	r^2
Con–9	.23	162	.34	.65	−20	.98
Con–5	.00	123	.00	.35	128	.97
Con–8	.60	55	.99	1.00	−60	1.00
Con–6	.17	86	.96	.60	88	1.00
Adv–1	.85	27	1.00	.60	145	1.00
Adv–6	.10	90	.96	.58	140	.99
Adv–2	.53	49	1.00	.75	72	.99
Adv–5	.20	57	.96	.49	167	.99
Sig–1	.45	0	1.00	.32	352	1.00
Sig–8	.17	92	.60	.68	125	.96
Sig–12	.22	84	.41	.75	102	.98
Sig–4	.53	14	.96	.79	111	1.00

	Empirical, Turn			Empirical, Distance		
Subject	S	I	r^2	S	I	r^2
Con–9	.06	13	.01	.38	178	.43
Con–5	.10	23	.07	.33	280	.33
Con–8	1.05	−13	.75	.74	173	.74
Con–6	.38	21	.35	.54	211	.62
Adv–1	.50	19	.44	.76	168	.74
Adv–6	.29	26	.17	.51	264	.50
Adv–2	.61	16	.73	.70	169	.75
Adv–5	.35	40	.30	.43	231	.53
Sig–1	1.03	28	.71	.52	57	.64
Sig–8	.12	30	.08	.68	248	.65
Sig–12	.44	17	.43	.67	288	.59
Sig–4	1.08	12	.83	.64	106	.84

	Predicted, Turn			Predicted, Distance		
Subject	S	I	r^2	S	I	r^2
Con–9	.14	13	.20	.56	129	.61
Con–5	.17	21	.60	.29	302	.66
Con–8	.73	2	.97	.92	36	.87
Con–6	.41	21	.80	.49	232	.78
Adv–1	.60	11	.92	.69	206	.98
Adv–6	.34	25	.74	.45	309	.75
Adv–2	.62	14	.97	.70	164	.95
Adv–5	.48	31	.80	.36	244	.92
Sig–1	.62	42	.95	.35	140	.60
Sig–8	.36	20	.82	.57	312	.78
Sig–12	.42	19	.84	.63	276	.80
Sig–4	.92	18	.91	.63	102	.98

TABLE 3

Correlations over Subjects between Model and Data with respect to Response Functions for Turn and Distance, for Each of the Two Models

	4-parameter model (no noise)	6-parameter model (noise)
Slope of Distance Function	.81	.81
Slope of Turn Function	.91	.92
Intercept of Distance Function	.77	.76
Intercept of Turn Function	.79	.81
r^2 of Distance Function	.72	.94
r^2 of Turn Function	.75	1.00

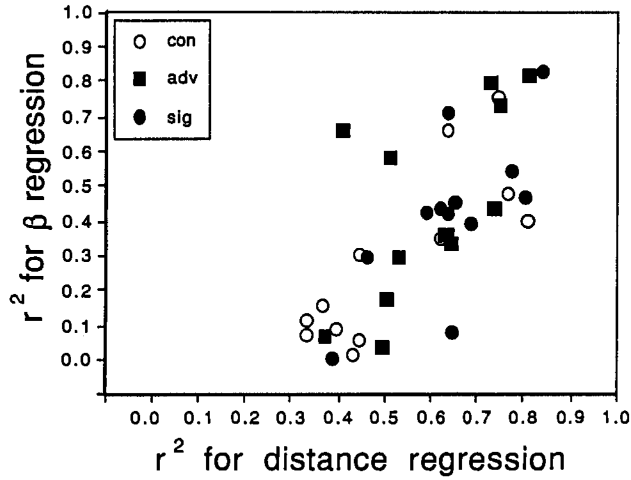

FIG. 7. The x and y Axes Represent Variance Accounted for by Linear Trend (r^2) in the Empirical Response Function for Distance and Turn-angle (β), Respectively. Each data point represents one subject; the points are coded to indicate the subject's blindness status.

empirical response function. There are six correlations, corresponding to the slope, intercept, and degree of linearity (r^2) of the response function for turn and distance. These correlations are generally over .7; thus the model accounts for about 50 percent of the variability among the subjects in the form of the response function.

Additional Sources of Variability

The degree of linearity (r^2) of the empirical response function seems to be a potentially important measure for differentiating subjects. Subjects with high linearity in the turn function also tend to show high linearity in the distance function; the correlation over all thirty-seven subjects between r^2 values of the response functions for turn and distance is .72, as shown in Figure 7.[3] The model preserves this relationship: For the twelve subjects that were simulated, the correlation between the predicted r^2 values for turn and distance is .61.[4]

Although the model preserves the relationship between the degree of linearity in the response functions, it does not fare well in predicting the actual value of r^2 for those functions. Table 2 shows that the model systematically predicts a higher degree of linearity in the relationship between subjects' responses and the correct values than the data actually show. That is, the r^2 values for the empirical response functions are consistently less than those for the model-predicted equivalents. This trend suggests that there is a source of random noise in the data that reduces the degree of linear fit relative predictions.

[3]The correlation between empirical response functions is greater with respect to the r^2 than the slopes (correlation = .45) or intercepts (−.089). These differences are preserved in the model's prediction for the present twelve subjects. The correlations between predicted response functions for turn and distance is .61 for r^2, .50 for slopes, and .37 for intercepts.

[4]We note a trend in the results indicating why a correlation between turn and distance performance might occur. Both turns and distances are based on the same encoded values for A, B, and α. Subjects who tend to encode segment lengths and turns with less compression (that is, the slopes of the encoding functions approach 1) tend to produce higher linearity in the response functions for both turn and distance. For the twelve subjects fit to the model, the correlation between the total of the encoding-function slopes and the r^2 in the empirical response function is .68 for turn and .84 for distance; for the predicted response functions the corresponding correlations are .48 for turn and .67 for distance.

Accordingly, we added a Gaussian noise parameter to the model's predictions of distance and turn. To the initially predicted response of the subject was added a value randomly generated from a Gaussian distribution with mean zero and a given standard deviation, the latter constituting the noise parameter. Because the randomness of the added noise produced substantial variations in the predicted response functions from run to run, simulated data were averaged from one hundred runs with the same noise parameter. There was a noise parameter for distance and one for turn. The parameters were determined by searching in the range .00–.50 in .01 increments and selecting noise levels so that the simulated response function matched the corresponding data function with respect to r^2 (with secondary consideration to matching the slope and intercept). Table 4 shows the predicted response functions when this Gaussian noise has been added, by subject, along with the noise values used to produce each function.

Table 3 further compares the original four-parameter model to the model when Gaussian noise has been added. The models are compared with respect to the same six correlations described above in evaluating the noise-free model. There is little difference between the two models' ability to predict the slope and intercept of the empirical response function. This is as we intended; the addition of random noise to the system should not disturb the underlying relation between the predicted and correct response. However, random noise reduces the strength of covariation in the predicted response function, simulating the noisiness of the data and improving the match between predicted and empirical r^2 values (compare Table 4 with Table 2, middle).

These simulations serve to demonstrate that noisiness in the response functions can result from a random process imposed on the more regular underlying encoding process. The precise locus of the random noise is unspecified by the simulation. Although we have added noise subsequent to encoding, which might be taken to simulate random perturbation in the computation of the response or in its output, it remains possible that there is a stochastic component to encoding itself.

One might expect that the noise parameters are greatest for subjects having the lowest value of r^2 in the empirical response function. In fact, this is not the case. The correlation between the noise parameter and empirical r^2, over subjects, is a reasonable .73 for turn, but only .10 for distance. In the latter case, the relationship is essentially U-shaped. Subjects whom the model determines to have the least overall compression in encoding (that is, those with the highest slopes for the hypothesized encoding functions, shown at the top of Table 3) tend to generate a high predicted r^2. If the empirical r^2 is also high (for example, Subject Sig-4),

TABLE 4

Predicted Response Functions for the Model with Added Noise. Noise parameter, slope, intercept, and r^2 for turn and distance, by subject (labeled as in Table 2).

	Turn				Distance			
Subject	noise	S	I	r^2	noise	S	I	r^2
Con–9	.28	.12	14	.06	.07	.56	129	.56
Con–5	.17	.16	21	.14	.10	.29	306	.38
Con–8	.16	.75	1	.76	.13	.93	29	.75
Con–6	.20	.39	22	.35	.09	.49	233	.62
Adv–1	.26	.59	11	.43	.13	.70	203	.74
Adv–6	.24	.33	26	.23	.13	.45	311	.48
Adv–2	.15	.62	13	.71	.15	.69	166	.67
Adv–5	.30	.48	31	.27	.11	.36	245	.51
Sig–1	.14	.62	41	.73	.01	.35	140	.60
Sig–8	.37	.34	22	.14	.09	.57	310	.66
Sig–12	.19	.41	20	.39	.13	.64	271	.62
Sig–4	.13	.93	17	.82	.10	.64	97	.80

little additional noise is needed to simulate it. But if the empirical value is somewhat lower (for example, Subjects Adv-1, Con-8, and Adv-2), considerable noise is required.

DISCUSSION

The encoding-error model is quite successful in accounting for the pattern of errors in triangle completion. It does so by assuming a systematic error pattern in encoding, by which we mean the set of processes that leads to a functional internal representation of the pathway and precedes computation of the desired response. For most subjects, the segment lengths and turns in the pathway appear to be compressed in the representation relative to their true values. The model assumes that this compression in encoding is the only source of systematic error; however, random error could arise at any stage in processing. In modeling individual subjects, it was necessary to incorporate a stochastic noise parameter into the model in order to account for the degree of nonsystematic variability. The randomness of this variability can be seen in the fact that when subjects were averaged, the model achieved an excellent fit without any assumption of noise.

In contrast to its assumptions about encoding, the model assumes that computation of the response and its execution are free of systematic error. Computation of the response is implemented in the model by the use of trigonometric formulas. Of course, this is not meant to imply that subjects are consciously performing mental trigonometric calculations. Successful computation might be achieved by an internal scanning process, performed on a spatial image, that derives the direction and distance of the origin. Alternatively, the computation might be a more abstract process that takes as its input nonspatial, even symbolic, values of segment lengths and turn extents. From the present model we cannot distinguish between such alternatives.

The model indicates that performance on relatively simple pathways can be accounted for without assuming error in stages other than encoding. It would, of course, be possible to add assumptions of systematic error in other stages, in which case subjects' performance would reflect the totality of error. However, such assumptions would add unnecessary degrees of freedom when accounting for the data from triangular pathways.

It appears, however, that additional assumptions would be needed to account for pathway completion using stimuli more complex than triangles. The model failed to provide as good an account of data from such stimuli. We have noted the extreme variability in responses with a pathway where one segment crossed over another (Klatzky et al. 1990; Loomis et al. 1993). It is in precisely these cases, which subjects find most difficult, that the encoding-error model itself breaks down. Clearly, there are sources of error in processing complex pathways that the model fails to capture. One likely source is the computation of the homeward trajectory, which may be affected by higher-level cognitive processes when the pathway is complex, particularly when subjects are judging its configural properties (for example, crossovers).

How might we understand the strong tendency toward regression to the mean in the encoding functions derived by the model? At a descriptive level, this regression means that the effective value of the stimulus (angle or segment length) was a compromise between the presented value and the mean of presented values, as averaged over the preceding trials. Such a compromise is a rational response, given uncertainty about the actual value of the stimulus. In the present task, people had to sense self-motion through tactual, proprioceptive, and vestibular cues, when passively led through the stimulus pathway. The sensory information was then presumably used to build a trace of the route and a survey representation. It is reasonable to suppose that subjects had considerable uncertainty both in sensing the path and building higher-level representations. Our meaning of uncertainty encompasses sensory noise and memory loss, as well as subjective awareness of lack of knowledge.

This view suggests that there should be greater regression toward the mean, the less effectively the navigator is able to encode information about a pathway. If it is applied to the data of Loomis et al. (1993), the implication is that groups differing by visual experience did not differ in uncertainty about the pathways they had navigated.

Were subjects given even minimal visual information about self-motion, by this reasoning, they should exhibit less regression. Work of Hayashi, Fujii, and Inui (1990) illustrates this point. They studied subjects' encoding of distances that were simulated in a computer graphics system, where optical flow and elapsed time were cues to distance traveled. The exponent of the power function for magnitude estimation was used as a measure of the quality of the subject's encoded distance value. Essentially, this exponent is a measure of regression to the mean (once scaling is used to match the response mean to that of the input): the lower the value, the greater the subject's tendency to underestimate long distances and to overestimate short ones. The exponent was found to decrease with memory load and to increase with practice on a given course.

However the effective information about a pathway is reduced, whether by restricting the stimulus information or by limiting its registration by the perceiver, the present model assumes that it is the ensuing encoded representation that limits performance in the triangle completion task. It is able to account for systematic errors in completing simple paths without further assumptions about errors from the computation of responses or their execution. Further research is needed to delineate the conditions under which the model applies, but the present application suggests at least that fallibility at the level of encoding is a potentially potent source of navigation error.

APPENDIX 1

The equation used for parameter estimation is derived as follows, where a, b, and $\underline{\alpha}$ are the internal encoded counterparts of segment A, segment B, and α in the stimulus, and i, j, and k are subscripts depicting particular values of these variables, c and $\underline{\beta}$ represent the measured values of the subject's responses, n is a subscript for the configuration and carat($\hat{}$) represents the predicted values.

When the values of a_i, b_j, and $\underline{\alpha}_k$ are given, the shape of the triangle (configuration n) is determined. That is, by the law of cosines,

$$\hat{c}_n = \left(a_i^2 + b_j^2 - 2a_i b_j \cos \underline{\alpha}_k\right)^{1/2}.$$

Also by the law of cosines,

$$\cos \hat{\underline{\beta}}_n = \left(b_j^2 + c_n^2 - a_i^2\right)/(2b_j c_n)$$

so that

$$\hat{\beta}_n = \cos^{-1}\left(\left(b_j^2 + c_n^2 - a_i^2\right)/(2b_j c_n)\right).$$

Using the above equations, we can compute the theoretical location of the endpoint in each configuration. Let the coordinates of the endpoint in configuration n be $\hat{x}_n$ and $\hat{y}_n$, in a coordinate system where the origin (0, 0) is the endpoint of segment B and x axis is aligned with segment B:

$$\hat{x}_n = \hat{c}_n \cos \underline{\hat{\beta}}_n,$$

$$\hat{y}_n = \hat{c}_n \sin \underline{\hat{\beta}}_n.$$

Also let the coordinates of the empirical location of the endpoint be x_n and y_n in configuration n:

$$x_n = c_n \cos \underline{\beta}_n,$$

$$y_n = c_n \sin \underline{\beta}_n.$$

The squared difference between the two locations (dif_n^2) is given by

$$dif_n^2 = (x_n - \hat{x}_n)^2 + (y_n - \hat{y}_n)^2.$$

Our task is to obtain six parameter values that minimize the following value, which sums the squared differences over the twenty-seven configurations.

$$S = \sum_{n=1-27} dif_n^2.$$

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